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# ON VECTOR-VALUED LIAPUNOV FUNCTIONS AND STABILIZATION OF INTERCONNECTED SYSTEMS

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V. D. FURASOV
(Moscow)

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We examine certain questions related to the application of vector-valued Liapunov functions to the solving of the stabilization problem for interconnected systems. We find the set of controls satisfying a damping criterion for the transient responses. We show that the proposed construction of the vector-valued Liapunov function enables us to solve the problem posed under milder constraints imposed on the controls than the constraints which can be obtained by Bailey's construction [1]. We determine the properties of the control set obtained. From the set found we select the controls ensuring the stabilization of interconnected systems under inclomplete information of the characteristics of the executive devices. We show the optimality of the controls selected with respect to the vector-valued functionals.

**1. Statement of the problem.** We consider a collection of $l$ smooth dynamic systems whose perturbed motion in the region

$$\| x_i \| \leqslant r_i, \quad t \geqslant 0 \quad (\| x_i \|^2 = (x_i, x_i), \quad r_i = \text{const}, \quad r_i > 0) \tag{1.1}$$

is described by the differential equations

$$x_i = F_i(x_i, t) + G_i(x_i, t) u_i + \sum_{j=1}^{l} H_{ij}(x_1, \ldots, x_l, t) x_j, \quad i = 1, \ldots, l \tag{1.2}$$

Here $x_i$ is the vector of the variables with respect to which the $i$ th system is stabilized, $x_i \in R^{n_i}$, $F_i$ and $G_i$ is a vector-valued function and a matrix, defined and continuous in (1.1) together with their partial derivatives in $x_i$ and $t$, $u_i$ is the control vector, $u_i \in R^{m_i}$, $H_{ij}$ is a matrix characterizing the influence of the $j$th system on the behavior of the $i$th system $(H_{ii} \equiv 0)$, $t$ is time. In analogy with [2, 3] we state the problem of stabilizing interconnected systems in the following way.

The problem. Given the positive numbers $h_{ij}$, $t_i^*$, $d_i$ and $\varepsilon_i (h_{ii} = 0, \varepsilon_i \leqslant d_i, \quad i, j = 1, \ldots, l)$, the collection of interconnected systems (1.2), the set $\Omega$ of continuous controls

$$u_i = u_i(x_i, t), \quad i = 1, \ldots, l$$

From $\Omega$ select a subset of controls, on which the specified collection of systems is asymptotically stable and the transient responses satisfy the condition

$$\| x_i(t) \| \leqslant \varepsilon_i, \quad t \geqslant t_i^*, \quad i = 1, \ldots, l \tag{1.3}$$

under any initial perturbations

$$\| x_i(0) \| \leqslant d_i, \quad i = 1, \ldots, l \tag{1.4}$$

and any matrices $H_{ij}$ satisfying the estimates

$$\| H_{ij}(x_1, \ldots, x_l, t)\, x_j \| \leqslant h_{ij} \| x_j \|, \quad i, j = 1, \ldots, l$$

Conditions (1.3), (1.4), which we call the damping criterion of the transient responses, have features in common with the problems of optimizing vector-valued functions [4–6] and with the problems considered in [3, 7, 8]. Therefore, the solution being proposed for the problem posed is based on Liapunov functions satisfying estimates typical of quadratic forms [9] and essentially related to the notion of a vector-valued Liapunov function [10–14].

**2. Solution of the problem.** Let $\beta_1, \ldots, \beta_l$ be positive constants, $V_1(x_1, t), \ldots, V_l(x_l, t)$ be Liapunov functions satisfying the estimates

$$c_{i1}^2 \| x_i \|^2 \leqslant V_i(x_i, t) \leqslant c_{i2}^2 \| x_i \|^2 \quad (c_{i1}, c_{i2} = \text{const} > 0, \ i = 1, \ldots, l) \tag{2.1}$$

$$\| \partial V_i / \partial x_i \|^2 \leqslant c_{i3}^2 V_i(x_i, t) \quad (c_{i3} = \text{const} > 0, \ i = 1, \ldots, l) \tag{2.2}$$

in region (1.1), $\lambda_1(x_1, t), \ldots, \lambda_l(x_l, t)$ be arbitrary smooth scalar functions, $P_1(x_1, t), \ldots, P_l(x_l, t)$ be arbitrary skew-symmetric matrices, $y_1^*(t), \ldots, y_l^*(t)$ be a particular solution of the equations

$$y_i^{\cdot} = -\frac{\beta_i}{2} y_i + \sum_{j=1}^{l} \frac{c_{i3} h_{ij}}{2 c_{j1}} y_j, \quad y_i^*(0) = c_{i2} d_i, \quad i = 1, \ldots, l \tag{2.3}$$

Theorem. The set of controls

$$u_i = -\frac{\lambda_i}{2} G_i' \frac{\partial V_i}{\partial x_i} + P_i G_i' \frac{\partial V_i}{\partial x_i}, \quad i = 1, \ldots, l \tag{2.4}$$

satisfies the transient response damping criterion if

$$\frac{\partial V_i}{\partial t} \leqslant -\beta_i V_i(x_i, t) - \left( \frac{\partial V_i}{\partial x_i}, F_i \right) + \frac{\lambda_i}{2} \left( \frac{\partial V_i}{\partial x_i}, G_i G_i' \frac{\partial V_i}{\partial x_i} \right), \quad i = 1, \ldots, l \tag{2.5}$$

the trivial solution of Eqs. (2.3) is asymptotically stable ($y_i^*(t) \to 0$ as $t \to \infty$) and

$$y_i^*(t) \leqslant c_{i1} \varepsilon_i, \quad t \geqslant t_i^*, \quad i = 1, \ldots, l \tag{2.6}$$

To prove the theorem we consider the derivatives of the functions $V_1(x_1, t), \ldots, V_l(x_l, t)$, defined on the motions of systems (1.2), corresponding to the controls (2.4). When the inequalities indicated are satisfied

$$V_i^{\cdot}(x_i, t) \leqslant \Delta(V_i(x_i, t))$$

$$\Delta(V_i) = -\beta_i V_i + \sum_{j=1}^{l} \frac{c_{i3} h_{ij}}{c_{j1}} (V_i V_j)^{1/2}, \quad i = 1, \ldots, l$$

Consequently, for all $t \geqslant 0$

$$V_i(x_i(t), t) \leqslant \xi_i(t), \quad i = 1, \ldots, l \tag{2.7}$$

if $V_i(x_i(0), 0) = \xi_i(0)$ and $\xi_i(t)$ is a particular solution of the equations

$$\xi_i^{\cdot} = \Delta(\xi_i), \quad i = 1, \ldots, l \tag{2.8}$$

In the case being considered $\xi_i(t) \neq 0$, if $\xi_i(0) \neq 0$; therefore, the change of variables $\xi_i = y_i^2$ reduces system (2.8) to the form (2.3). The trivial solution of the equations obtained is asymptotically stable by virtue of the theorem's hypotheses. Consequently, the trivial solution of Eqs. (1.2), (2.4) also is asymptotically stable.

Further, under any initial perturbations (1.4)

$$(V_i (x_i (t), t))^{1/2} \leqslant y_i^* (t), \quad t \geqslant 0, \ i = 1, \ldots, l$$

and since here $\| x_i \| \leqslant (V_i (x_i, t))^{1/2} / c_{i1}$, for all $t \geqslant 0$ we have

$$\| x_i (t) \| \leqslant y_i^* (t) / c_{i1}, \quad i = 1, \ldots, l$$

and by virtue of condition (2.6) of the theorem inequalities (1.3) are fulfilled on all motions of systems (1.2), (2.4) stating in region (1.4). The theorem is proved.

Let us now show that the construction of the vector-valued Liapunov function used to prove the theorem leads to solving the problem posed under constraints on the controls milder than the constraints obtained by the vector-valued Liapunov function proposed by Bailey [1]. To do this we consider inequalities (2.1), (2.2) and (2.5). When they are satisfied

$$V_i^{\cdot} (x_i, t) \leqslant -\beta_i V_i (x_i, t) + c_{i3} (V_i (x_i, t))^{1/2} \sum_{j=1}^{l} h_{ij} \| x_j \|, \quad i = 1, \ldots, l$$

Hence in accord with [1]

$$V_i^{\cdot} (x_i, t) \leqslant -\frac{\beta_i}{2} V_i (x_i, t) + \frac{1}{2\beta_i} c_{i3}^2 \sum_{j=1}^{l} h_{ij}^2 \sum_{j=1,\, j \neq i}^{l} \| x_j \|^2 \leqslant \Delta^* (V_i(x_i, t))$$

$$\Delta^* (V_i) = -\frac{\beta_i}{2} V_i + \frac{1}{2\beta_i} c_{i3}^2 \sum_{j=1}^{l} h_{ij}^2 \sum_{j=1,\, j \neq i}^{l} \frac{1}{c_{j1}^2} V_j, \quad i = 1, \ldots,$$

Thus, for all $t \geqslant 0$

$$V_i (x_i (t), t) \leqslant z_i (t), \quad i = 1, \ldots, l \tag{2.9}$$

if $V_i(x_i (0), 0) = z_i (0)$ and $z_i^{\cdot} = \Delta^* (z_i)$, $i = 1, \ldots, l$. But in the case being considered

$$\Delta (V_i) \leqslant -\frac{\beta_i}{2} V_i + \frac{1}{2\beta_i} \Big[ \sum_{j=1}^{l} \frac{c_{i3} h_{ij}}{c_{j1}} (V_j)^{1/2} \Big]^2 \leqslant \Delta^* (V_i), \ i = 1, \ldots, l$$

Consequently, for all $t \geqslant 0$

$$\xi_i (t) \leqslant z_i (t), \quad i = 1, \ldots, l$$

which proves the assertion made (see estimates (2.7) and (2.9)).

We note that when $h_{ij} = 0$, $i, j = 1, \ldots, l$, according to the theorem, the set of controls (2.4) satisfy the transient response damping criterion if

$$\beta_i \geqslant \frac{1}{t_i^*} \ln \frac{c_{i2} d_i}{c_{i1} \varepsilon_i}, \quad i = 1, \ldots, l$$

The application of the estimates (2.9), however, yields in this case

$$\beta_i \geqslant \frac{2}{t_i^*} \ln \frac{c_{i2} d_i}{c_{i1} \varepsilon_i}, \quad i = 1, \ldots, l$$

**3. Determination of the properties of the controls obtained.** The control set (2.4) is defined to within the arbitrary functions $\lambda_1 (x_1, t), \ldots, \lambda_l (x_l, t)$ and matrices $P_1 (x_1, t), \ldots, P_l (x_l, t)$. It is natural that when solving actual stabilization problems the arbitrariness in the choice of the functions and matrices named can be used in the most different ways. However, here we need to keep in mind that when the $\lambda_i (x_i, t) \leqslant 0$, the transient responses in systems (1.2) undoubtedly satisfy the specified performance criterion with $u_i \equiv 0$ if they satisfy this performance criterion by virtue of the hypotheses of the theorem proved.

Taking the following inequalities as satisfied

$$\lambda_i (x_i, t) > 0 \tag{3.1}$$

we consider the controls

$$u_i = -\frac{\lambda_i}{2} G_i' \frac{\partial V_i}{\partial x_i} \tag{3.2}$$

and we assume that

$$\frac{\partial V_i}{\partial t} = -\beta_i V_i (x_i, t) - \left(\frac{\partial V_i}{\partial x_i}, F_i\right) + \frac{\lambda_i}{2} \left(\frac{\partial V_i}{\partial x_i}, G_i G_i' \frac{\partial V_i}{\partial x_i}\right), \quad i = 1, \ldots, l \tag{3.3}$$

Controls (3.2) are, obviously, a special case of controls (2.4). Here, if (2.4) is the control set ensuring the fulfillment of the inequalities

$$\left(\frac{\partial V_i}{\partial x_i}, F_i + G_i u_i\right) + \frac{\partial V_i}{\partial t} \leqslant -\beta_i V_i (x_i, t), \qquad i = 1, \ldots, l$$

in region (1.1), then (3.2) are controls guaranteeing the fulfillment of the inequalities indicated with the smallest value of the quantity $z_w = \| u_1 \|^2 + \ldots + \| u_l \|^2$ at each point $\{x_1, \ldots, x_l, t\}$ of this region. By analogy with [3, 8] controls (3.2) are called constraint-optimal controls.

Constraint-optimal controls possess important property: they are the solution of the stabilization problem for the systems

$$x_i = F_i (x_i, t) + G_i (x_i, t) \varphi_i (u_i, t) + \sum_{j=1}^{l} H_{ij} (x_1, \ldots, x_l, t) x_j, \qquad i = 1, \ldots, l \tag{3.4}$$

for any continuous vector-valued functions $\varphi_1 (u_1, t), \ldots, \varphi_l (u_l, t)$, satisfying the inequalities

$$(u_i, \varphi_i (u_i, t)) \geqslant (u_i, u_i), \quad t \geqslant 0, \qquad i = 1, \ldots, l \tag{3.5}$$

In fact, when inequalities (3.5) are satisfied

$$\left(\frac{\partial V_i}{\partial x_i}, F_i + G_i \varphi_i \left(-\frac{\lambda_i}{2} G_i' \frac{\partial V_i}{\partial x_i}, t\right)\right) + \frac{\partial V_i}{\partial t} = -\beta_i V_i (x_i, t) +$$
$$\left(\frac{\partial V_i}{\partial x_i}, G_i \varphi_i \left(-\frac{\lambda_i}{2} G_i' \frac{\partial V_i}{\partial x_i}, t\right)\right) + \frac{\lambda_i}{2} \left(\frac{\partial V_i}{\partial x_i}, G_i G_i' \frac{\partial V_i}{\partial x_i}\right) \leqslant -\beta_i V_i (x_i, t)$$
$$i = 1, \ldots, l$$

and the validity of the assertion made follows immediately from the theorem proved.

Krasovskii and Letov have established the connection between Liapunov functions and optimal control problems. Of additional interest here is the fact that constraint-optimal controls are controls optimal also with respect to the vector-valued functional

$$I_i(u_1, \ldots, u_l) = \int_0^\infty \left[\beta_i V_i (x_i, t) - \sum_{j=1}^{l} \left(\frac{\partial V_i}{\partial x_i}, H_{ij} x_j\right) - \right. \tag{3.6}$$
$$\left. \frac{\lambda_i}{4} \left(\frac{\partial V_i}{\partial x_i}, G_i G_i' \frac{\partial V_i}{\partial x_i}\right) + \sum_{j=1, j \neq i}^{l} \left\| u_j + \frac{\lambda_j}{2} G_j' \frac{\partial V_j}{\partial x_j} \right\|^2 + \frac{1}{\lambda_i} \| u_i \|^2 \right] dt$$
$$i = 1, \ldots, l$$

In fact, in accordance with [15] the controls

$$u_k^\circ = -\frac{\lambda_k}{2} G_k' \frac{\partial S}{\partial x_k}, \quad u_j^\circ = -\frac{\lambda_j}{2} G_j' \frac{\partial V_j}{\partial x_j} - \frac{\lambda_j}{2} G_j' \frac{\partial S}{\partial x_j}, \; j \neq k, j = 1, \ldots, l \tag{3.7}$$

impart the minimum of the $k$th functional in (3.6) if the trivial solution of Eqs. (1.2), (3.7) is asymptotically stable and the function $S = S(x_1, \ldots, x_l, t)$ satisfies a partial differential equation. This equation can be satisfied by setting $S = V_k(x_k, t)$. Then

$$u_i^\circ = -\frac{\lambda_i}{2} G_i' \frac{\partial V_i}{\partial x_i}, \qquad i = 1, \ldots, l \tag{3.8}$$

and since the trivial solution of Eqs. (1.2), (3.8) is asymptotically stable by virtue of the construction of controls (3.2), the validity of the assertion made follows directly from [15].

The construction of constraint-optimal controls is related to the solving of the partial differential equations (3.3). However, controls (3.2) are also a solution of the problem posed when the Liapunov functions satisfy inequalities (2.5). When the inequalities indicated and inequalities (3.1) are fulfilled, controls (3.2) are called quasioptimal controls. The properties of quasioptimal controls are analogous to the properties of constraint-optimal controls. Thus, on all motions of system (1.2), starting in region (1.4), they impart the minimum of the functionals

$$I_i(u_1, \ldots, u_l) = \int_0^\infty \Big[\frac{\lambda_i}{2}\Big(\frac{\partial V_i}{\partial x_i},\ G_i G_i' \frac{\partial V_i}{\partial x_i}\Big) - \Big(\frac{\partial V_i}{\partial x_i},\ F_i\Big) - \frac{\partial V_i}{\partial t} -$$

$$\sum_{j=1}^{l}\Big(\frac{\partial V_i}{\partial x_i},\ H_{ij}x_j\Big) + \sum_{j=1,\ j\neq i}^{l}\Big\| u_j + \frac{\lambda_j}{2} G_j \frac{\partial V_j}{\partial x_j}\Big\|^2 + \frac{1}{\lambda_i}\| u_i \|^2\Big] dt, \qquad i = 1, \ldots, l$$

When inequalities (3.5) are fulfilled, quasioptimal controls are a solution of the problem posed for systems (3.4).

**4. Example.** Let the perturbed motion of interconnected systems be described by the differential equations

$$x_i = A_i x_i + B_i \varphi_i(u_i, t) + \sum_{j=1}^{l} H_{ij}(x_1, \ldots, x_l, t)\, x_j$$

$$\operatorname{rank}\,[B_i, A_i B_i, \ldots, A_i^{n_i - 1} B_i] = n_i, \qquad i = 1, \ldots, l$$

where $A_i$ and $B_i$ are constant matrices satisfying the condition indicated. In this case, having set $\lambda_i = \mathrm{const} > 0$, the theorem's hypotheses can be satisfied by the functions

$$V_1 = (x_1, \Gamma_1 x_1), \ \ldots, \ V_l = (x_l, \Gamma_l x_l) \tag{4.1}$$

The solving of the problem posed now reduces to seeking matrices $\Gamma_1, \ldots, \Gamma_l$ satisfying the equations

$$0 = -\beta_i \Gamma_i - \Gamma_i A_i - A_i' \Gamma_i + 2\lambda_i \Gamma_i B_i B_i' \Gamma_i \tag{4.2}$$

or the inequalities

$$0 \leqslant -\beta_i \Gamma_i - \Gamma_i A_i - A_i' \Gamma_i + 2\lambda_i \Gamma_i B_i B_i' \Gamma_i, \qquad i = 1, \ldots, l$$

to determining the smallest and largest eigenvalues of the matrices found, and to verifying the fulfillment of condition (2.6).

When the condition indicated is fulfilled, the controls

$$u_i = -\lambda_i B_i' \Gamma_i x_i, \qquad i = 1, \ \ldots, \ l$$

generated by (4.1) satisfy the transient response damping criterion under any variations of the vector-valued functions $\varphi_1(u_1, t), \ldots, \varphi_l(u_l, t)$, admissible by inequalities (3.5).

Let us now consider certain peculiarities of the solution of Eqs. (4.2) when $A_i$, $B_i$ are $(n \times n)$- and $(n \times 1)$-matrices

$$A_i = \begin{Vmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{Vmatrix}, \quad B_i = \begin{Vmatrix} 0 \\ 0 \\ \dots \\ 0 \\ 1 \end{Vmatrix}, \quad i = 1, \dots, l$$

In the case being considered, when $n = 2$ Eqs. (4.2) can be satisfied by the matrices

$$\Gamma_i = \frac{1}{2\lambda_i} \begin{Vmatrix} \beta_i^3 & \beta_i^2 \\ \beta_i^2 & 2\beta_i \end{Vmatrix}, \quad i = 1, \dots, l \quad (\lambda_1 > 0, \dots, \lambda_l > 0)$$

where $\lambda_1, \dots, \lambda_l$ are arbitrary constants satisfying the inequalities indicated. For $n = 3$ the matrices $\Gamma_i$ are determined by the expressions

$$\Gamma_i = \frac{1}{2\lambda_i} \begin{Vmatrix} \beta_i^5 & 2\beta_i^4 & \beta_i^3 \\ 2\beta_i^4 & 5\beta_i^3 & 3\beta_i^2 \\ \beta_i^3 & 3\beta_i^2 & 3\beta_i \end{Vmatrix}, \quad i = 1, \dots, l$$

Hence, by induction, for any $n$

$$\Gamma_i = \frac{1}{2\lambda_i} \begin{Vmatrix} \gamma_{11}^{\circ}\beta_i^{2n-1} & \gamma_{12}^{\circ}\beta_i^{2n-2} & \dots & \gamma_{1,n-1}^{\circ}\beta_i^{n+1} & \gamma_{1n}^{\circ}\beta_i^{n} \\ \gamma_{12}^{\circ}\beta_i^{2n-2} & \gamma_{22}^{\circ}\beta_i^{2n-3} & \dots & \gamma_{2,n-1}^{\circ}\beta_i^{n} & \gamma_{2n}^{\circ}\beta_i^{n-1} \\ \dots & \dots & \dots & \dots & \dots \\ \gamma_{1,n-1}^{\circ}\beta_i^{n+1} & \gamma_{2,n-1}^{\circ}\beta_i^{n} & \dots & \gamma_{n-1,n-1,n}^{\circ}\beta_i^{3} & \gamma_{n-1,n}^{\circ}\beta_i^{2} \\ \gamma_{1n}^{\circ}\beta_i^{n} & \gamma_{2n}^{\circ}\beta_i^{n-1} & \dots & \gamma_{n-1,n}^{\circ}\beta_i^{2} & \gamma_{nn}^{\circ}\beta_i \end{Vmatrix} \qquad (4.3)$$

where, and this is essential, $\gamma_{ij}^{\circ}$ are positive constants not dependent on $\beta_i, i = 1, \dots, l$.

We now assume that by applying various methods, the solution of Eqs. (4.2) can be obtained on a computer. Then, solving one of Eqs. (4.2) with $\beta_i = 1$, $\lambda_i = 1$ and next substituting the found values of $\gamma_{ij}^{\circ}$ into (4.3), we obtain the matrices $\Gamma_1, \dots, \Gamma_l$ corresponding to any values of constants $\beta_1, \dots, \beta_l$.

The example considered shows quite intuitively the possibility of combining analytic and computer methods for constructing interconnected systems having a specified performance. Besides, in the case being considered the matrices $\Gamma_i$ can be found purely analytically for any $n$.

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## TWO-DIMENSIONAL FLOWS OF A RELAXING MIXTURE AND THE STRUCTURE OF WEAK SHOCK WAVES

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A. L. NI and O. S. RYZHOV
(Moscow)

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Plane-parallel and axisymmetric flows of a chemically active mixture in which only a single reaction takes place are considered on the assumption that the equilibrium and the frozen speeds of sound in the medium are nearly equal. The asymptotic system of equations which in the nonlinear theory of small perturbations is valid in the range of transonic speeds is used. An exact particular solution of these equations is derived, which makes it possible to trace the process of shock wave onset and development. If the particle velocity is higher than the equilibrium but lower than the frozen speeds of sound, the shock waves are totally dispersed, as in the case of one-dimensional flows. Waves containing discontinuities with incomplete dispersion are generated, if the stream velocity exceeds the frozen speed of sound.